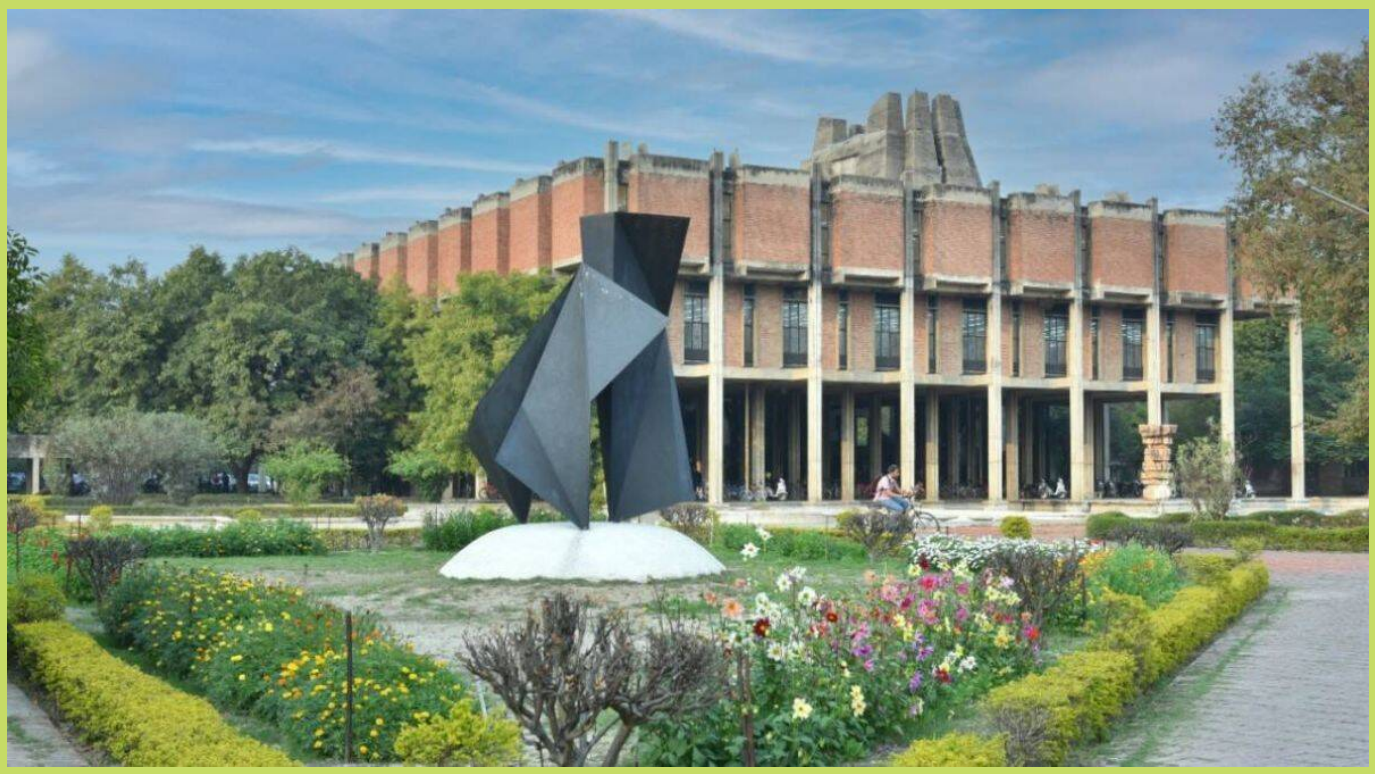
Annual Departmental Open House - 2023



February 04, 2023

Department of Mathematics & Statistics
Indian Institute of Technology Kanpur
Kanpur - 208016

Coordinator: Prof. Debasis Kundu

Annual Departmental Open House - 2023

Timing

9:30 AM - 5:30 PM

Venue

L-16, Lecture Hall Complex, IIT Kanpur

Organized By

Department of Mathematics & Statistics
Indian Institute of Technology Kanpur
Kanpur - 208016

Contact

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Department of Mathematics & Statistics
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Session Chairs

Prof. Amit Kuber
Prof. Indranil Chowdhury
Prof. Malay Banerjee
Prof. Ashis Mandal

Volunteers

Abhinek Shukla (18108261)
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Sanjay Kumar (19108276)
Arvind Kumar Nath (19208264)
Ritesh Kumar Pandey (19208266)
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Mrs. Sarita Nigam & Mr. Subhash & Mr. Mukesh

Programme Overview

Inauguration *Time: 9:30 AM - 9:35 AM*

Prof. Mohua Banerjee, HoD

Title of Talks

Session - 01 *Time: 9:35 AM - 10:55 AM*

Chair : Prof. Amit Kuber

S01:01 Multipliers of the Hilbert spaces of Dirichlet series.
Speaker: Chaman Kumar Sahu
Timing: 9:35 AM - 9:55 AM

S01:02 Classification of irreducible Harish-Chandra modules for loop extended Witt algebras.
Speaker : Ritesh Kumar Pandey
Timing: 9:55 AM - 10:15 AM

S01:03 Filling with separating curves.
Speaker : Bhola Nath Saha
Timing: 10:15 AM - 10:35 AM

S01:04 Tate cohomology and Base change of cuspidal representations of GL_n.
Speaker : Sabyasachi Dhar
Timing: 10:35 AM - 10:55 AM

Tea and Coffee Break *Time: 10:55 AM - 11:15 AM*

Session - 02

Time: 11:15 AM - 12:55 PM

Chair : Prof. Indranil Chowdhury

S02:01 The Generalized Pitman Nearness Criterion for order-restricted parameters.
Spaeker : Naresh Garg
Timing: 11:15 AM - 11:35 AM

S02:02 On computationally efficient estimators of a two-dimensional chirp model with the quadratic phase.
Speaker : Abhinek Shukla
Timing: 11:35 AM - 11:55 AM

S02:03 Estimation methods for elementary chirp model parameters.
Speaker : Anjali Mittal
Timing: 11:55 AM - 12:15 PM

S02:04 Multivariate strong invariance principle for Markov chain Monte Carlo.
Speaker : Arka Banerjee
Timing: 12:15 PM - 12:3 5 PM

S02:05 Testing Homological Equivalence Using Betti Numbers.
Speaker : Satish Kumar
Timing: 12:35 PM - 12:55 PM

Session - 03

Time: 2:15 PM - 3:35 PM

Chair : Prof. Malay Banerjee

S03:01 Age-dependent immuno-epidemiological model with distributed recovery and death rates.
Speaker : Samiran Ghosh
Timing: 2:15 PM - 2:35 PM

S03:02 von Neumann's inequality for the n-dimensional Hartogs triangle.
Speaker : Shubham Jain
Timing: 2:35 PM - 2:55 PM

S03:03 Bifurcations of spatio-temporal patterns and continuation of bifurcating branches.
Speaker : Subrata Dey
Timing: 2:55 PM - 3:15 PM

S03:04 L´evy Flows and associated Stochastic PDEs.

Speaker : Arvind Kumar Nath

Timing: 3:15 PM - 3:35 PM

Tea and Coffee Break *Time: 3:35 PM - 3:55 PM*

Session - 04

Time: 3:55 PM - 5:15 PM

Chair : Prof. Ashis Mandal

S04:01 Monoids related to self-homotopy equivalences of fibred product.

Speaker : Gopal Chandra Dutta

Timing: 3:55 PM - 4:15 PM

S04:02 Operator theory on generalized Hartogs triangles.

Speaker : Paramita Pramanick

Timing: 4:15 PM - 4:35 PM

S04:03 A two-dimensional moment problem.

Speaker : Rajkamal Nailwal

Timing: 4:35 PM - 4:55 PM

S04:04 Inhomogeneous Khintchine-Groshev type theorems on manifolds over function fields.

Speaker : Sourav Das

Timing: 4:55 PM - 5:15 PM

Certificate Ceremony

Time: 5:15 PM - 5:30 PM

Abstracts

Session - 01 *Time: 9:35 AM - 10:55 AM*

Chair : Prof. Amit Kuber

S01:01 **Speaker** : Chaman Kumar Sahu

Title: Multipliers of the Hilbert spaces of Dirichlet series.

Abstract: Given a RKHS $\mathcal{H}$ on a non-empty set X, a function $\phi : X \to \mathbb{C}$ is called multiplier if $\phi \cdot f \in \mathcal{H}$ for all $f \in \mathcal{H}$.

For a sequence $\mathbf{w} = \{w_j\}_{j=2}^{\infty} \subseteq (0, \infty)$, consider the positive semi-definite kernel $\kappa_{\mathbf{w}}(s, u) = \sum_{j=2}^{\infty} w_j j^{-s-\overline{u}}$ defined on some right-half plane $\mathbb{H}_\rho$ for a real number ρ. In our recent work, we obtained a sufficient condition on $\mathbf{w}$ that classifies the multiplier algebra of Hilbert space $\mathscr{H}_{\mathbf{w}}$ associated with $\kappa_{\mathbf{w}}$. As an application, we describe the multiplier algebra when $\mathbf{w}$ is an additive function satisfying certain inequalities.

S01:02 **Speaker** : Ritesh Kumar Pandey

Title: Classification of irreducible Harish-Chandra modules for loop extended Witt algebras.

Abstract: The representation theory of infinite dimensional Lie algebras has received a lot of attention in recent decades due to its significance in both mathematics and physics.

Let A_n be a Laurent polynomial ring in $n(\geq 1)$ variables then the Lie algebra of derivations of A_n is denoted by W_n or $\mathrm{Der}(A_n)$ and is called a Witt algebra. All the irreducible modules for W_n with finite-dimensional weight spaces have been classified by Billig and Futorny, they turn out to be either uniformly bounded or "highest weight" modules. Let S be any finite-dimensional abelian Lie algebra over $\mathbb{C}$. The newly made Lie algebra $\mathfrak{L}_{S,n} := W_n \ltimes (S \otimes A_n)$ is called an extended Witt algebra. Let B be any finitely generated commutative associative algebra with unity over $\mathbb{C}$, then the Lie algebra $\mathfrak{L}_{S,n}(B) := \big(W_n \ltimes (S \otimes A_n)\big) \otimes B$ is called a loop extended Witt algebra with the brackets: $[u_1 \otimes b_1, u_2 \otimes b_2] = [u_1, u_2] \otimes b_1 b_2$ for $u_i \in \mathfrak{L}_{S,n}$ and $b_i \in B$, $i = 1, 2$.

In this presentation, we will see, the classification of irreducible Harish-Chandra modules for loop-extended Witt algebras. They turn out to be either modules with uniformly bounded weight spaces or highest-weight modules. We further see that all these modules are single-point evaluation modules ($n \geq 2$). So they are actually irreducible modules for extended Witt algebras.

This is joint work with Prof. Sachin S. Sharma (thesis supervisor), Dr. Priyanshu Chakraborty, and Prof. Eswara Rao Senapathi.

S01:03 **Speaker** : Bhola Nath Saha

Title: Filling with separating curves.

Abstract: A pair (α, β) of simple closed curves on a closed and orientable surface S_g of genus g is called a filling pair if the complement is a disjoint union of topological discs. If α is separating, then we call it as separating filling pair. We find a necessary and sufficient condition for existence of a separating filling pair on S_g with exactly two complementary discs. We study the combinatorics of the action of the mapping class group Mod(S_g) on the set of such filling pairs. Furthermore, we construct a Morse function $\mathcal{F}_g$ on the moduli space $\mathcal{M}_g$ which, for a given hyperbolic space X, outputs the length of shortest such filling pair with respect to the metric in X. We show that the cardinality of the set of global minima of the function $\mathcal{F}_g$ is same as the number of Mod(S_g)-orbits of such filling pair.

S01:04 **Speaker** : Sabyasachi Dhar

Title: Tate cohomology and Base change of cuspidal representations of GL_n**.**

Abstract : Let p and l be distinct primes and let n be a positive integer such that l and p does not divide n. Let F be a finite extension of $\mathbb{Q}_p$, and let E be a finite Galois extension of degree l of a p-adic field F. Let π and ρ be two l-adic integral smooth cuspidal representations of $\mathrm{GL}_n(E)$ and $\mathrm{GL}_n(F)$ respectively such that π is obtained as base change of ρ. Then the Tate cohomology $\widehat{H}^0(\pi)$, as an l-modular representation of $\mathrm{GL}_n(F)$, is well defined. In this talk, we will show that if l is banal for $GL_{n-1}(F)$, the representation $\widehat{H}^0(\pi)$ is isomorphic to the Frobenius twist of the reduction mod-l of the representation ρ. This is based on joint work with Santosh Nadimpalli.

Tea and Coffee *Time: 10:55 AM - 11:15 AM*

Session - 02

Time: 11:15 AM - 12:55 PM

Chair : Prof. Indranil Chowdhury

S02:01 **Speaker** : Naresh Garg

Title: The Generalized Pitman Nearness Criterion for order-restricted parameters.

Abstract: We consider component-wise estimation of order-restricted location parameters of a general bivariate location distribution under the generalized Pitman nearness criterion (GPN). We develop some general results that, in many situations,

are useful in finding improvements over location equivariant estimators. In particular, under certain conditions, these general results provide improvements over the unrestricted Pitmannearest location equivariant estimators and restricted maximum likelihood estimators. The usefulness of the obtained results is illustrated through their applications to specific probability models. A simulation study has been considered to compare how well different estimators perform under the GPN criterion with a specific loss function.

S02:02 **Speaker** : Abhinek Shukla

Title: On computationally efficient estimators of a two-dimensional chirp model with the quadratic phase.

Abstract: We address the problem of parameter estimation of a real-valued two dimensional (2D) chirp signal model, contaminated with linear stationary errors. The parameter estimation problem for the chirp model is encountered in many real-life applications such as 2D-homomorphic signal processing, magnetic resonance imaging (MRI), optical imaging, interferometric synthetic aperture radar (INSAR) etc. In recent times, several methods have been proposed for parameter estimation of these models. These methods however are either statistically sub-optimal and suffering with high signal-to-noise ratio (SNR) threshold, or computationally burdensome (e.g., least squares estimators (LSEs)). We propose estimators that are computationally faster than the conventional optimal methods such as LSEs, and at the same time, having desirable statistical properties such as, attaining same rates of convergence as the optimal LSEs. We will also discuss more theoretical results obtained for these estimators along with some simulation results comparing performances of different estimators.

S02:03 **Speaker** : Anjali Mittal

Title: Estimation methods for elementary chirp model parameters.

Abstract: We propose some estimation methods to estimate the elementary chirp model parameters, which are encountered in sonar, radar, acoustics, and other areas. We derive asymptotic theoretical properties of least squares estimators (LSEs) and approximate least squares estimators (ALSEs) for the one-component elementary chirp model. It is proved that the proposed estimators are strongly consistent and follow the normal distribution asymptotically. We also suggest how to obtain proper initial values for these methods. The problem of finding initial values is a difficult problem when the number of components in the model is large, or when the signal-to-noise ratio is low, or when two frequency rates are close to each other. We propose sequential procedures to estimate the multiple-component elementary chirp model parameters. We prove that asymptotic properties of the sequential LSEs and sequential ALSEs coincide with the LSEs and ALSEs, respectively. Real data

analysis on a bat echolocation data is performed, illustrating the performance of the proposed sequential algorithm and validating the capability of the model to capture such real datasets.

S02:04 **Speaker** : Arka Banerjee

Title: Multivariate strong invariance principle for Markov chain Monte Carlo.

Abstract: Strong invariance principle (SIP) for the partial sums of some mean-corrected random sequences holds if it can be approximated to a scaled Brownian motion in an almost everywhere sense. Over the years SIP rates have consistently improved for various dependent and independent structures of the random sequences. In this paper, we provide relatively tight SIP rates for polynomially and geometrically ergodic Markov chains. We use wide-sense regenerative structures of Harris ergodic Markov chains, bypassing the need to assume a one-step minorization. A key feature of the rates is that they are completely specified. This allows users of the Markov chain Monte Carlo to verify key assumptions employed in output analysis.

S02:05 **Speaker** : Satish Kumar

Title: Testing Homological Equivalence Using Betti Numbers.

Abstract: In this talk, I shall be discussing testing the homological equivalence of two spaces based on the random samples generated from these spaces. We shall see that one can use the one and two-sample tests based on the Betti numbers to classify two spaces underlying the data.

Session - 03

Time: 2:15 PM - 3:35 PM

Chair : Prof. Malay Banerjee

S03:01 **Speaker** : Samiran Ghosh

Title: Age-dependent immuno-epidemiological model with distributed recovery and death rates.

Abstract: To better understand the spread of infectious diseases in populations, we need to realize that each infected individual harbors the pathogen and that pathogen is in dynamic interplay with the host immune system. The spread of diseases on a population level depends on these within-host dynamics of infected individuals. Moreover, the immunological condition varies from one individual to another, which can influence the disease progression on the population level. To take into account these aspects, we consider an age-dependent immuno-epidemiological model with distributed recovery and death rates. Disease transmission rate depends on the intra-subject pathogen load determined from the immunological submodel. The

age-dependent model includes the pathogen load, recovery and death rates as functions of age considered as a continuous variable. Modeling results show that initial age distribution of the population can influence the epidemic progression significantly, depending upon the rates of infectivity and susceptibility of different age groups.

S03:02 **Speaker** : Shubham Jain

Title: von Neumann's inequality for the n-dimensional Hartogs triangle.

Abstract: We define a Hardy space $H^2(\triangle_0^n)$ on the n-dimensional Hartogs triangle

$$\triangle_0^n := \{(z_1, z_2, \ldots, z_n) \in \mathbb{C}^n : |z_1| < |z_2| < \ldots < |z_n| < 1\}.$$

We then discuss an analogue of von Neumann's inequality for $\triangle_0^n$. (This talk is based on joint work with Sameer Chavan and Paramita Pramanick).

S03:03 **Speaker** : Subrata Dey

Title: Bifurcations of spatio-temporal patterns and continuation of bifurcating branches.

Abstract: Spatial pattern formation plays a crucial role in understanding the dynamical complexity of biological systems. Turing instability is a well-known mechanism behind the formation of stationary spatial patterns by reaction-diffusion system in population biology. At first, we consider a prey-predator model with cooperative hunting among predators modelled by a Holling type-II functional response having a saturated encounter rate [1]. Bifurcation analysis is carried out on the temporal model to detect local bifurcations such as transcritical, saddle-node, Hopf, Bogdanov-Takens bifurcations and global homoclinic bifurcation is identified. Main objective of this talk is to discuss the existence of various stationary Turing patterns within a bounded domain and their bifurcations through numerical continuation. For suitable values of parameters, spatio-temporal chaos via period-doubling cascade is observed when a given parameter traverses the Turing-Hopf region.

Reference: Subrata Dey, Malay Banerjee, and S. Ghorai."Bifurcation Analysis and Spatio-Temporal Patterns of a Prey–Predator Model with Hunting Cooperation." International Journal of Bifurcation and Chaos 32.11 (2022): 2250173.

S03:04 **Speaker** : Arvind Kumar Nath

Title: L´evy Flows and associated Stochastic PDEs.

Abstract: We first explore certain structural properties of L´evy flows and use this information to obtain the existence of strong solutions to a class of Stochastic PDEs in the space of tempered distributions, driven by L´evy noise. The uniqueness of the solutions follows from Monotonicity inequality. These results extend an earlier

work on the diffusion case.

Tea and Coffee *Time: 3:35 PM - 3:55 PM*

Session - 04

Time: 3:55 PM - 5:15 PM

Chair : Prof. Ashis Mandal

S04:01 **Speaker** : Gopal Chandra Dutta

Title: Monoids related to self-homotopy equivalences of fibred product.

Abstract: We introduce a nested sequence of monoids related to self-homotopy equivalences of fibrewise pointed spaces, such that the limit is the group of homotopy classes of fibrewise pointed self-equivalences. We explore this monoid for fibred product in terms of individual spaces. Further we study two related invariants associated to these monoids: self-closeness number and self-length.

S04:02 **Speaker** : Paramita Pramanick

Title: Operator theory on generalized Hartogs triangles.

Abstract: We consider the family $\mathcal{P}$ of n-tuples P consisting of polynomials $P_1, \ldots, P_n$ with nonnegative coefficients which satisfy $\partial_i P_j(0) = \delta_{i,j}, i, j = 1, \ldots, n$. With any such P, we associate a Reinhardt domain $\triangle_P^n$ that we call the generalized Hartogs triangle. We are particularly interested in the choices $P_a = (P_{1,a}, \ldots, P_{n,a})$, $a \geq 0$, where $P_{j,a}(z) = z_j + a\prod_{k=1}^n z_k,\ j = 1, \ldots, n$. The generalized Hartogs triangle associated with P_a is given by

$$\triangle_a^n = \Big\{ z \in \mathbb{C}\times\mathbb{C}_*^{n-1} : |z_j|^2 < |z_{j+1}|^2(1-a|z_1|^2),\ j = 1, \ldots, n-1, |z_n|^2+a|z_1|^2 < 1 \Big\}.$$

The domain $\triangle_0^2$ is the Hartogs triangle. Unlike most domains relevant to the multivariable operator theory, the domain $\triangle_P^n$, $n \geq 2$, is never polynomially convex. There is a reproducing kernel Hilbert space $\mathscr{H}_m^2(\triangle_P^n)$ of holomorphic functions associated with every generalized Hartogs triangle $\triangle_P^n$. We study the space $\mathscr{H}_m^2(\triangle_P^n)$ and the multiplication n-tuple $\mathscr{M}_z$ acting on $\mathscr{H}_m^2(\triangle_P^n)$. This is a joint work with S. Chavan and S. Jain.

S04:03 **Speaker** : Rajkamal Nailwal

Title: A two-dimensional moment problem.

Abstract: Given a polynomial $p : [0, \infty) \times [0, \infty) \to (0, \infty)$, we address the 2-dimensional Hausdorff moment problem of finding a positive Borel measure μ_p

concentrated on $[0,1] \times [0,1]$ such that

$$\frac{1}{p(m,n)} = \int_{[0,1]\times[0,1]} s^m t^n d\mu_p(s,t), \quad m,n \geq 0.$$

In the talk we discuss the problem for polynomials $p(x,y)$ of the form $b(x)+a(x)y$, where a, b are polynomials with roots $-a_j$, $-b_j$ in $(-\infty, 0)$ such that $b_1 \leq a_1 \leq b_2 \leq \ldots \leq b_k \leq a_k$. As a consequence, we show that for $p(x,y) = a+bx+cy+dxy$, $\{\frac{1}{p(m,n)}\}_{m,n\geq 0}$ is a minimal completely monotone net if and only if $ad - bc \leq 0$.

The talk is based on joint work with Akash Anand and Sameer Chavan.

S04:04 **Speaker** : Sourav Das

Title: Inhomogeneous Khintchine-Groshev type theorems on manifolds over function fields.

Abstract: We will talk about metric Diophantine approximation and its connection with homogeneous dynamics. A complete Khintchine-Groshev type theorem in both homogeneous and inhomogeneous setting, on analytic nondegenerate manifolds over a local field of positive characteristic will be discussed. The main ingredient in the proof of homogeneous convergence case is a dynamical technique due to Bernik, Kleinbock and Margulis. The inhomogeneous transference principle and the technique of ubiquitous systems are the keys to proving the inhomogeneous Khintchine-Groshev theorems. This is based on joint work with Arijit Ganguly.