Grade Table (for checker use only)

Question	Points	Score
1	8	
2	10	
3	14	
4	14	
5	18	
6	18	
7	22	
8	26	
9	30	
10	30	
Total:	190	

Team Members (Name and Roll no):

- ....
- ....
- ....

INSTRUCTIONS:

- **Write your team name on top of each page.**
- If you have any queries, contact an invigilator. Any sort of interaction with another team can lead to a penalty or disqualification.
- Submit any electronic devices that you possess, to one of the invigilators. You may collect them after the event. Any team caught using any electronic device will be immediately disqualified.
- Enough space has been provided in the question paper. Use it wisely. However, if you need extra sheets, contact an invigilator.

1. (8 points) Define the function $f(x, y, z)$ by

$$f(x, y, z) = x^{y^z} - x^{z^y} + y^{z^x} - y^{x^z} + z^{x^y}$$

Evaluate $f(1, 2, 3) + f(1, 3, 2) + f(2, 3, 1) + f(2, 1, 3) + f(3, 1, 2) + f(3, 2, 1)$

2. (10 points) Start with n equally spaced dots $P_1, P_2, ...P_n$ on a straight line , with a distance 1 between consecutive dots. Using P_1P_2 as a base side, draw a regular pentagon in the plane. Similarly, draw $n - 2$ additional regular pentagons on the base side P_1P_3, P_1P_4, ..., P_1P_n, all pentagons lying on the same side of the the line P_1P_n. Find the total number of dots.

3. (14 points) An urn contains 1729 balls of different colors. Randomly select a pair, repaint the first to match the second, and replace the pair in the urn. What is the expected time until the balls are all the same color?

4. (14 points) In trapezoid ABCD, with sides AB and CD parallel, $\angle DAB = 6°$ and $\angle ABC = 42°$. Point X on side AB is such that $\angle AXD = 78°$ and $\angle CXB = 66°$. If AB and CD are 1 unit apart, Find AD + DX- BC - CX

5. (18 points) An immortal flea jumps on whole points of the number line, beginning with 0. The length of the first jump is 3, the second 5, the third 9, and so on. The length of k^{th} jump is equal to $2^k + 1$. The flea decides whether to jump left or right on its own. Is it possible that sooner or later the flea will have been on every natural point, perhaps having visited some of the points more than once?

6. (18 points) 100 integers are arranged in a circle. Each number is greater than the sum of the two subsequent numbers (in a clockwise order). Determine the maximal possible number of positive numbers in such circle.

7. (22 points) Prove that $\dfrac{[(2+\sqrt{3})^{2n-1}]-1}{2}$ is a perfect square for all $n \in N$. Where $[x]$ is the greatest integer function

8. (26 points) Let a and b be positive integers such that $a! + b!$ divides $a!b!$. Prove that $3a \geq 2b + 2$.

9. (30 points) Let $\mathbb{R}$ be the set of real numbers. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ that satisfy the equation

$$f(x + f(x + y)) + f(xy) = x + f(x + y) + yf(x)$$

for all real numbers x and y.

10. (30 points) Prove that

$$|\cos(x)|+|\cos(y)|+|\cos(z)|+|\cos(y+z)|+|\cos(z+x)|+|\cos(x+y)|+3|\cos(x+y+z)| \geq 3$$

for all real x, y, and z.