Grade Table (for checker use only)

Question	Points	Score
1	10	
2	5	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	15	
Total:	100	

Team Members:

- ....
- ....
- ....

INSTRUCTIONS:

- **Write your team name on top of each page.**
- If you have any queries, contact an invigilator. Any sort of interaction with another team can lead to a penalty or disqualification.
- Submit any electronic devices that you possess, to one of the invigilators. You may collect them after the event. Any team caught using any electronic device will be immediately disqualified.
- Enough space has been provided in the question paper. Use it wisely. However, if you need extra sheets, contact an invigilator.

1. (10 points) Two IITK alumni (Mr. Ashu and Mr. Apurv) bump into each other after over 20 years from their graduation.
   Ashu: "How have you been?"
   Apurv: "Great! I got married and I have three daughters now."
   Ashu: "Really? How old are they?"
   Apurv: "Well, the product of their ages is 72, and the sum of their ages is the same as the number on that building over there."
   Ashu: "Right, ok.. oh wait..hmm, I still don't know."
   Apurv: "Oh sorry, the oldest one just started to play the piano."
   Ashu: "Wonderful! My oldest is the same age!"
   What are the ages of Apurv's daughters?

2. (5 points) 50 integers are chosen among the first 100 positive integers $(1, 2, ..., 100)$ in such a manner that no two of the chosen integers have a sum equal to 100. Show that there is at least one perfect square among the numbers chosen.

3. (10 points) Points on a plane are coloured either *red* or *blue*. Prove that

   (a) there is a line segment whose mid point and end points are all of the same colour.

   (b) there is an equilateral triangle whose vertices are all of the same colour.

4. (10 points) In a ΔABC, the median from A is perpendicular to the median from B. If $BC = 7$ and $AC = 6$, find AB.

5. (10 points) Find all positive real numbers x, y, z such that

$$2x - 2y + \frac{1}{z} = \frac{1}{2017}, \quad 2y - 2z + \frac{1}{x} = \frac{1}{2017}, \quad 2z - 2x + \frac{1}{y} = \frac{1}{2017}.$$

6. (10 points) In a ΔABC, M is the mid-point of BC, P is any point on AM and PE, PF are perpendiculars to AB, AC respectively. If EF is parallel to BC, find out the value of $\angle A$.

7. (10 points) In an acute angled triangle ABC, prove that $\sin A + \sin B + \sin C > 2$.

8. (10 points) Let $d(n)$ denote the number of positive divisors of the positive integer n. Determine those numbers n for which $d(n^3) = 5d(n)$.

9. (10 points) Is is possible for a knight to start on any square of a $4 \times n$ chessboard, visit every square exactly once and return back to its original square?

10. (15 points) Two players A and B take turns removing chips from a pile that initially contains n chips. The first player A cannot remove all the n chips at the beginning. A player, on his turn, must remove at least one chip. However he cannot remove more chips than those his opponent removed on his previous turn. The player who removes the last chip is the winner. Assuming both players play optimally, give the condition on n such that B always wins.