Bianchi Type I Anisotropic Universe with $\omega < -1$ without Big Smash

Anil Kumar Yadav

Anand Engineering College, Agra-282 007, India

ABSTRACT

A spatially homogeneous and anisotropic Bianchi Type I universe has been studied with $\omega < -1$ without Big Smash. It is demonstrated that if cosmic dark energy behaves like a fluid with equation of state $p = \omega\rho$ (p and ρ being pressure and energy density respectively) as well as generalized chaplygin gas simultaneously, Big Rip or Big Smash problem does not arise even for equation of state parameter $\omega < -1$ unlike other phantom models, here the scale factor for Bianchi Type I universe is found regular for all time.

Big Rip : Big rip is a cosmological hypothesis (2003) about the ultimate fate of universe in which the matter of universe from stars and galaxies to atom and subatomic particles are progressively torn apart by the expansion of universe at a certain time in future.

ABSTRACT

The present model is derived from Bianchi Type I space-time equation using the effective role of GCG behaviour in a natural way.

Key words: Dark energy matter, Big Smash and Bianchi Type I Universe.

INTRODUCTION and MOTIVATIONS

One of the most important properties of FRW models is, as predicated by the inflation, the flatness, which agrees with observed cosmic microwave background radiation. Even through the universe on large scale, appear homogeneous and isotropic at the present time, there is no observational data that guarantee in an epoch prior to the recombination. In the early universe the sorts of matter fields are uncertain.

Inflation: Exponential expansion of early universe

INTRODUCTION and MOTIVATIONS

The existence of anisotropy at early times is a natural phenomenon to investigate, as an attempt to clarify among other things, the local anisotropies that we observe today in galaxies, cluster and super clusters so at early time it appears appropriate to suppose a geometry that is more general than just the isotropy and homogeneous FRW geometry.

INTRODUCTION and MOTIVATIONS

A Bianchi Type I model, being the straightforward generalization of the flat FRW model, is one of the simplest models of the anisotropic universe that describes homogeneous and spatially flat universe.

INTRODUCTION and MOTIVATIONS

Unlike FRW space-time which has the same scale factor for each of the three spatial directions, Bianchi Type I space-time has a different scale factor in each direction, thereby introducing an anisotropy to the system.

INTRODUCTION and MOTIVATIONS

The Bianchi type I space- time is given by

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \quad ...(1)$$

Where A, B and C are the metric coefficients.

Theoretically accelerated expansion of universe is obtained when the cosmological model is supposed to be dominated by a fluid obeying the equation of state $p = \rho\omega$ with p as isotropic pressure, ρ as energy density and $-1 \leq \omega < -1/3$.

INTRODUCTION and MOTIVATIONS

In the recent past, it was pointed out that the current data also allowed $\omega < -1$ [6] Rather, in refs. [7, 8, 9] it is discussed that these data favor $\omega < -1$ being EOS parameter for phantom dark energy. Analysis of recent Ia Supernova data support $\omega < -1$ strongly [10,11,12]

Phantom energy: it is the hypothetical form of dark energy with $\omega <$ -1. At $\omega < -1$, the universe will eventually be pulled apart.

INTRODUCTION and MOTIVATIONS

The idea is pursued in refs. [17,18,19] and shown that an escape from the Big-Smash is possible on making quantum corrections to energy density ρ and pressure p in Bianchi Type I space-time. In the framework of Robertson-Walker cosmology, Chaplygin gas (CG) is also consider as a good source of dark energy for having negative pressure, given as

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$$P = -\frac{A_0}{\rho} \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad ..(2)$$

With $A_0 > 0$, Moreover, It is only gas having super symmetry generalization [20, 21]. Bertolami et al [11] have found the generalized Chaplygin gas (GCG) is better fit for latest Supernova data. In the case of GCG, equation (2) looks like

$$p = -\frac{A_0}{\rho^{1/\alpha}} \ldots\ldots\ldots \quad (3)$$

Where $1 \leq \alpha < \infty.$

INTRODUCTION and MOTIVATIONS

For α =1 equation (3) corresponds to equation(2).

In this paper, a different prescription for GR based Future universe, dominated by the dark energy with $\omega < -1$, is proposed which is not leading to the catastrophic situations mentioned above. The scale factor, obtained here, does not possess future singularity.

INTRODUCTION and MOTIVATIONS

In the present model, it is assumed that the dark energy behaves like GCG, obeying equation (3) as well as fluid with equation of state

$$p = \omega\rho \,.......... \,..(4)$$

here ω < -1,

Connecting equation (3) with the hydrodynamic equation

$$\rho_4 = -3\frac{a_4}{a}(\rho + p)........ ....(5)$$

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And integrating, it is obtained that

$$\rho^{(1+\alpha)/\alpha} = A_0 + (\rho_0^{(1+\alpha)/\alpha} - A_0)(a_0/a)^{3(1+\alpha)/\alpha} ....(6)$$

where $\rho_0 = \rho(t_0)$, $a_0 = a(t_0)$ and t_0 is the present time.

Equation (3 and (4) yield ω as

$$\omega(t) = -\frac{A_0}{\rho^{(1+\alpha)/\alpha}} ....(7a)$$

So, evaluation of (7a) at $t = t_0$ leads to

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$$A_0 = -\omega_0 \rho^{(1+\alpha)/\alpha} \text{.........}(7b)$$

with $\omega_0 = \omega(t)$.

From equation (6) and (7), it is obtained that

$$\rho = \rho_0 \left[-\omega_0 + (1+\omega_0)\left(\frac{a_0}{a}\right)^{3(1+\alpha)/\alpha} \right]^{\frac{\alpha}{(1+\alpha)}} \text{.........}(8)$$

with $\omega_0 < -1$.

INTRODUCTION and MOTIVATIONS

In the homogeneous model of universe, a scalar field with potential V(Φ) has the energy density

$$\rho_\phi = \frac{1}{2}\phi_4{}^2 + V(\phi)\text{.........}\ ..(9a)$$

and pressure

$$p_\phi = \frac{1}{2}\phi_4{}^2 - V(\phi)\text{.........}\ ....(9b)$$

Using equation (3), (4), (7) and (8), it is obtained that

INTRODUCTION and MOTIVATIONS

$$\phi_4{}^2 = \frac{(1+\omega_0)\rho_0{}^{(1+\alpha)/\alpha}(a_0/a)^{3(1+\alpha)/\alpha}}{\left[-\omega_0 + (1+\omega_0)(a_0/a)^{3(1+\alpha)/\alpha}\right]^{\alpha/(1+\alpha)}} \ldots\ldots\ldots\ldots\ldots(10)$$

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This equation shows that $\phi_4^{\ 2} > 0$ (giving positive kinetic energy) for $\omega_0 > -1$, which is the case of quintessence and $\phi_4^{\ 2} < 0$ (giving negative kinetic energy) for $\omega_0 < -1$, being the case of super-quintessence. As a reference, it is relevant to mention that long back, Hoyle and Narlikar used C-field (a scalar called creation) with negative kinetic energy for steady state theory of the universe [26]. Thus, and it is shown that the dual behavior of dark energy fluid, obeying equation (3) and (4) is possible for scalars, frequently used for cosmological dynamics. So, this assumption is not unrealistic.

LAW OF VARIATION OF HUBBLE'S PARAMETER

we define, $a = (ABC)^{1/3}$ as the average scale factor so that the Hubble's parameter in anisotropic models may be defined as

$$H = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)..............(11)$$

also we have

$$H = \frac{1}{3}(H_1 + H_2 + H_3)..................(12)$$

where H_1, H_2 and H_3 are directional Hubble's factor in the direction of x, y and z- axis respectively.

LAW OF VARIATION OF HUBBLE'S PARAMETER

Very recently, S. Kumar and C. P. Singh [2] have investigated a spatially homogeneous and anisotropic Bianchi Type I model by applying a special law of variation of Hubble's parameter that yield a constant value of DP. The law of variation of Hubble's parameter is

$$H = Da^{-n} = D(ABC)^{-n} \quad \text{........(13)}$$

where D and n are positive constant.

LAW OF VARIATION OF HUBBLE'S PARAMETER

The deceleration parameter (q) is given by

$$q = -\frac{aa_{44}}{a_4^{\,2}} \ldots\ldots\ldots\ldots\ldots\ldots.(14)$$

From equation (11) and (13), we get

$$\frac{a_4}{a} = Da^{-n} \ldots\ldots\ldots\ldots\ldots(15)$$

Which on integration lead to

$$a = (nDt + c_0)^{\frac{1}{n}} \ldots\ldots\ldots\ldots(16)$$

LAW OF VARIATION OF HUBBLE'S PARAMETER

where c_0 is constant of integration.

Equation (16) is yielding accelerated expansion of universe with a(t) as t $\rightarrow \infty$, supporting observational evidences of Ia Supernova [22,23] and WMAP [24, 25]. It is interesting to see that expansion, obtained here, is free from "finite time future singularity" unlike other General Relativity based phantom models. It is due to GCG behavior of phantom dark energy.

LAW OF VARIATION OF HUBBLE'S PARAMETER

In this case, Hubble distance is given by

$$H^{-1} = nt + D_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

where $D_0 = c_0/D$ is constant

Equation (17) showing the growth of Hubble's distance with time such that $t \to 0, H^{-1} \to D_0$ and $t \to \infty, H^{-1} \to \infty$ This means that, in present case galaxies will not disappear when $t \to \infty$ unlike the phantom models with future singularity, where galaxies are expected to vanish near future singularity.

LAW OF VARIATION OF HUBBLE'S PARAMETER

The horizon distance for this case is obtained as

$$d_H = a(t)\int_0^t \frac{dt'}{a(t')} \text{..............}(18)$$

From equation (16) and (18), we have

$$d_H = \frac{1}{D(n-1)}\left[(nDt + c_0) - D'(nDt + c_0)^{\frac{1}{n}}\right] \text{.............}(19)$$

Equation (20) showing that

LAW OF VARIATION OF HUBBLE'S PARAMETER

Equation (19) showing that

$$d_H(t) > a(t) \quad \text{for} \quad t > \frac{(1+D')^{\frac{n}{n-1}} - c_0}{nD}$$

So, horizon grows more rapidly than scale factor implying colder and darker universe. It is like flat or open universe with dominance of dark energy.

LAW OF VARIATION OF HUBBLE'S PARAMETER

Using equation (8) and (16) the expression for energy density is given by

$$\rho = \rho_0 \left[-\omega_0 + (1+\omega_0) \left(\frac{a_0}{\left(nDt + c_0\right)^{\frac{1}{n}}} \right)^{\frac{3((1+\alpha)}{\alpha}} \right]^{\frac{\alpha}{1+\alpha}} \qquad \ldots\ldots\ldots\ldots\ldots(20)$$

DISCUSSION

The Bianchi type I anisotropic universe without big smash is driven by using the law of variation of Hubble's parameter. It is found that the dark energy behaves like a fluid as well as generalized chaplygin gas simultaneously, big smash problem do not arise unlike other phantom models.

DISCUSSION

It is unlike GR based models driven by equation of state $p = \omega\rho$, where $\omega < -1$ having future singularity at $t = t_s$, where ρ and p are divergent [7, 13] or ρ is finite and p is divergent [17, 27]. Based on Ia supernova data, Singh et al [12] have estimated ω_0 for model in the range $-2.4 < \omega_0 < -1.74$ up to 95% confidence level. Taking this estimate as an example with $\alpha = 3, \rho_\infty = \rho(t \to \infty)$ is found in the range $1.51\rho_0 < \rho_\infty < 1.92\rho_0$.

DISCUSSION

This does not yield much increase in ρ as $t \rightarrow \infty$ but if this model is realistic and future experiment support large $|\omega_0|$, ρ_∞ will be very high. In both cases, small or large value of ω_0 increases in ρ indicate creation of Phantom dark energy in future. It may be due to decay of some other components of energy in universe, which is not dominating, for example cold dark matter. It is interesting to see that Big smash problem does not arise in the present model. In refs [16, 17, 18, 19] for models with future singularity, escape from cosmic doomsday is demonstrated using quantum correction in field equations near .Here using classical approach, a model for phantom cosmology, with accelerated expansion, is explored with is free from catastrophic situations.

DISCUSSION

This model is derived from Bianchi type I space time using effective role of GCG behavior in natural way. Srivastav [28] have investigated FRW model with ω < -1 without Big Smash and found $1.15\rho_0 < \rho_\infty < 1.24\rho_0$ where as the present model is Bianchi Type I universe with ω < -1 without Big Smash and found $1.51\rho_0 < \rho_\infty < 1.92\rho_0$ which is very closed to recently Ia supernova data.

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