Ph.D Selection Test

Department of Physics

Indian Institute of technology Kanpur

December 9, 2019 Time: 9:00AM − 11:00AM Maximum marks: 70

Question 1

(a) Evaluate the following integral:

$$\oint_C \frac{e^{iz}}{z(z-\pi)} dz$$

for each of the following contours: [5]

(i) C is a circle of radius R where $R > \pi$.

(ii) C is a circle of radius R where $R < \pi$.

(b) Radioactive nuclei decay according to the law $\frac{dN}{dt} = -\lambda N$, with N and λ being the concentration and the decay constant of a given nuclide, respectively. In a radioactive series of two different nuclides, with concentrations $N_1(t)$ and $N_2(t)$, we have

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \ , \quad \text{and} \quad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \ .$$

Find $N_2(t)$ for the initial conditions $N_1(0) = N_0$ and $N_2(0) = 0$. [5]

Question 2

Fermion may occupy a given single-particle state with maximum occupation probability 1. A parafermion is a particle for which the maximum occupancy of any given single-particle state is k, which is a positive integer. Consider a system with one single-particle level whose energy is ϵ, i.e. the total energy $E = \epsilon N$, where N is the particle number. Consider the system to be coupled with a reservoir of chemical potential μ and temperature T.

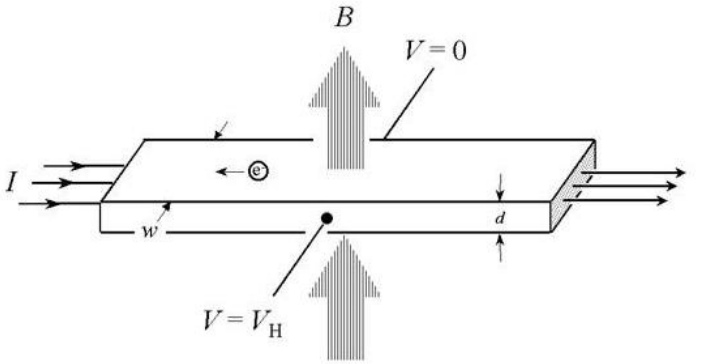
(a) Write down the the grand-partition function and find an expression of the average occupation number $\bar{N}$. [5]

(b) Considering $k = 3$, plot $\bar{N}$ as function of ϵ, clearly mentioning its value at $\epsilon = \mu, \epsilon \gg \mu$ and $\epsilon \ll \mu$. [5]

Question 3

(a) Consider the schematic given below demonstrating Hall effect. Write down the expression for Hall voltage (V_H) in terms of the applied current (I) and the magnetic field (B). If you want to design a Hall probe to measure an unknown magnetic field, would you use a metal

or semiconductor as the sensor? Whether a thin or thick sensor is desirable to enhance the sensitivity of the Hall probe? Justify your answers. [2+1+1]

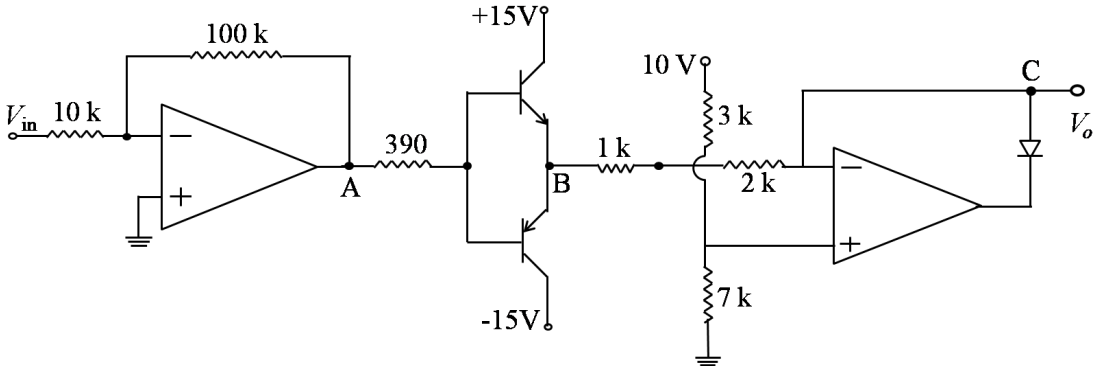


(b) What is the angular dispersion and resolving power of a diffraction grating? Would you use a grating or a prism for spectral analysis? Explain why. [2+1]

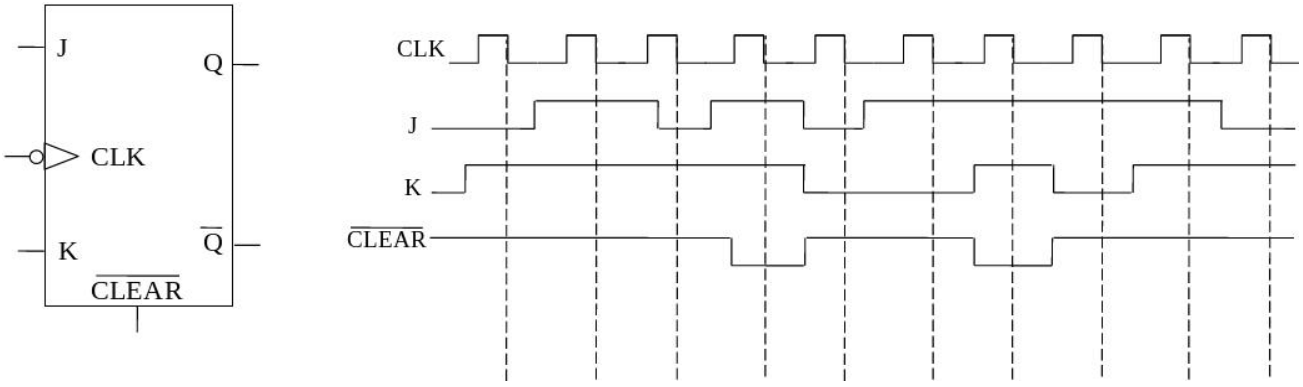
(c) What is the advantage of 4-probe measurement technique over 2-probe method? In which case the advantage becomes insignificant? [2+1]

Question 4

(a) Sine wave of 1 V amplitude and 25 kHz frequency has been applied at the inpur V_{in} of the circuit shown in the figure. Draw waveforms corresponding to nodes A, B and C along with the V_{in} input waveform. [5]



(b) Draw the time diagram for Q output of the shown negative edge triggered J-K flip-flop for the given input pulses. Assume that the flip-flop was SET initially. [5]



Question 5

Sketch a two-dimensional triangular lattice neatly and clearly. Assume that the length of each side of the triangle is d. Draw a primitive unit cell. Write down the basis vectors required to reproduce the whole crystal in 2D space and show them on the sketch. Calculate the reciprocal lattice vectors. [1+1+4+4]

Question 6

(a) The Lagrangian of a free particle of mass m moving with velocity $\mathbf{v}$, in the inertial frame K, is $L = \frac{1}{2}m\mathbf{v}^2$. Suppose another reference frame K' moves with a velocity $\mathbf{V}(t)$ with respect to frame K. Write down the Lagrangian L' in frame K'. Explain the dynamical origin of all the terms appearing in L', and specify whether you expect such a Lagrangian from your knowledge of pseudo-forces. [5]

(b) There is a phase space transformation given as

$$Q = \alpha q^a p\,, \quad P = \beta q^b\,,$$

where $a,\ b,\ \alpha,\ \beta$ are constants. Find out the values of a and b in terms of α and β if the above transformation is a canonical transformation. [5]

Question 7

(a) Consider an electron from rest is accelerated through a potential difference V. What is the de Broglie wavelength of the accelerated electron? (mass of the electron is m and charge e). [2]

(b) Suppose $\mid \alpha\rangle$ and $\mid \beta\rangle$ are two parity eigenstates. Show that $\langle\beta \mid \vec{r} \mid \alpha\rangle = 0$ unless the two states are of opposite parity. [3]

(c) Consider a two dimensional state space spanned by the basis states

$$|i\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |j\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} .$$

Suppose the Hamiltonian of a system is given as

$$H = \begin{pmatrix} 1 & \alpha \\ \alpha & 1 \end{pmatrix} ,$$

where α is a real constant. If the system starts out at $t = 0$ in state $|j\rangle$, what is its state at time t. [5]