<u>**Ph.D Selection Test**</u>

Department of Physics

Indian Institute of technology Kanpur

December 6, 2018 Time: 9:30AM − 11:30AM Maximum marks: 70

<u>Question 1</u>

Suppose a particle with mass m is moving in a two dimensional plane in presence of a potential V which only depends on $|\mathbf{r}| = r$ where $\mathbf{r}$ specifies the position of the particle.

(a) Write down the Lagrangian of the particle using plane polar coordinates. From the Lagrangian show that the angular momentum of the particle remains constant. [3 marks]

(b) The conserved energy function E is

$$E = \dot{q}^\alpha \frac{\partial L}{\partial \dot{q}^\alpha} - L\,,$$

where a summation over α is implicit. Here q^α are the generalized coordinates r and θ for $\alpha = 1, 2$. Evaluate the functional form of E in terms of r, θ and their time derivatives. Using the conservation of angular momentum write down E only in terms of r and $\dot{r}$. [3 marks]

(c) Define the radial effective potential as $V(r)$ plus all the terms in E which does not have any time derivative and call it $V'(r)$. Then write down E in terms of V' and other possible terms depending on $\dot{r}$. [2 marks]

(d) Write down explicitly the form of $V'(r)$ when $V(r) = -k/r$ where k is a positive, real constant. Schematically plot V' and show its minima. [2 marks]

<u>Question 2</u>

(a) Compute the integral on the principal branch

$$\int_0^\infty \frac{x^{k-1}}{a+x} dx$$

with $0 < k < 2$ and $a > 0$. [7 marks]

(b) Evaluate $\int_0^\infty \frac{x^{k-1}}{(x+1)^2} dx$, with $0 < k < 2$. [3 marks]

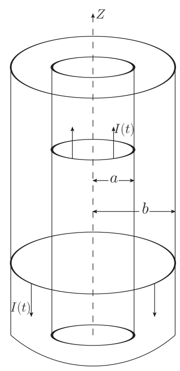
<u>Question 3</u>

Two infinitely long, thin cylindrical shells of radii a and b, placed coaxially with z axis, carry current $I(t) = I_0 \sin \omega t$ (on their surfaces) along $+\hat{z}$ (inner) and $-\hat{z}$ (outer) directions(see figure). Within the quasi-static approximation, obtain the magnitude and direction of:

(a) the magnetic field $\mathbf{B}$ for the three regions $\rho < a$, $a < \rho < b$, $\rho > b$, where ρ is the axial distance from z axis, [2 marks]

(b) the induced electric field $\mathbf{E}$ for $\rho < a$ [take $\mathbf{E}(\rho \to \infty) = 0$], [4 marks]

(c) the first correction $\mathbf{B}^{(1)}$ to the magnetic field for $\rho < a$. [4 marks]



Question 4

Consider a system of isolated N non- interacting particles. Each particle can only be in one of the two energy levels ϵ_0 and $-\epsilon_0$. The system is in contact with a heat reservoir at a temperature T.

(a) Derive the appropriate partition function of the system.[3 marks]

(b) Calculate the average energy of the system and plot it as a function of temperature. [4 marks]

(c) Plot the specific heat of the system as a function of temperature. [3 marks]

Question 5

Consider potential $V(x) \leq 0$ throughout the x-axis and $V(x \to \pm\infty) \to 0$. Let the corresponding Hamiltonian be denoted by H and let the true ground-state wavefunction for this potential be $\psi(x)$. Simultaneously consider a potential well of depth V_0 and of width such that $V(x)$ is below V_0 throughout the width of the well (see figure). Let us write this potential as $V_w(x)$. Note that we have made sure that $(V(x) - V_w(x)) \leq 0$ over the entire x-axis. Let the corresponding Hamiltonian, ground-state wavefunction and the eigenenergy be H_0, $\phi(x)$, and E_0, respectively.

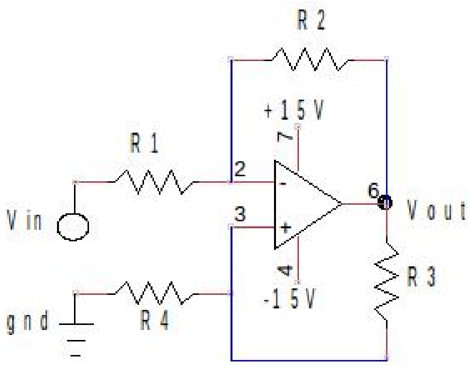
(i) What is the inequality satisfied by the expectation values $\langle \phi \mid H \mid \phi \rangle$ and $\langle \psi \mid H \mid \psi \rangle$? [3 marks]

(ii) Now write $H = H_0 - V_w(x) + V(x)$ and use the inequality in (i) to show that the ground-state energy of H is less than E_0. [4 marks]

(iii) Use a property of H_0 that it has at least one bound state to prove that a bound-state always exists for a particle in one-dimension in a potential that is negative throughout and vanishes as $x \to \pm\infty$. [3 marks]

Question 6

For the circuit shown below, derive an expression for the gain (V_{out}/V_{in}). [10 marks]



Question 7

Write your answers to three significant digits.

(a) On a casino table, 10 dice were rolled simultaneously.

(i) Write down an expression for probability of m of the dice to come up with the same digit. [3 marks]

(ii) Calculate the mean and standard deviation for the roll. [2 marks]

(b) The distribution of emitted particles is measured for a radioactive source in forward and backward directions. In one measurement of 1000 particles it was found that 415 particles were emitted in forward direction and 585 particles were emitted in backward direction.

(i) What is the uncertainty associated with each of these numbers, if the probabilities of emission are known to be equal in forward and backward directions. [3 marks]

(ii) If the probabilities are not equal, what uncertainty would you assign to the measurements using the above data? [2 marks]